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演習第九

Solve at least three of the following problems.

1. Prove or disprove the following:

   (1.1) $\mathcal{REG}$ is closed under inverse homomorphisms.

   (1.2) $\mathcal{CF}$ is closed under inverse homomorphisms.

2. Prove or disprove the following theorem.

   For all $L \in \mathcal{CF}$ and for all $R \in \mathcal{REG}$ we have $L \setminus R \in \mathcal{CF}$.

3. Consider the following language over the alphabet $\Sigma = \{0, 1, 2\}$

$$L = \{0^n 2 1^m \mid n, m \in \mathbb{N}, \ n \leqslant m \leqslant 4n\} .$$

   (3.1) Prove that L is context-free.

   (3.2) Construct a pushdown automaton $\mathcal{K} = [Q, \Sigma, \Gamma, \delta, q_0, k_0, F]$ such that $L(\mathcal{K}) = L$.

4. Prove the following: Let Σ be any alphabet such that $\text{card}(\Sigma) = 1$. Prove that

$$\mathcal{CF} \cap \wp(\Sigma^*) = \mathcal{REG} \cap \wp(\Sigma^*) ,$$

   i.e., every context-free language $L \subseteq \Sigma^*$ is regular.