On the Power of Monotonic Language Learning *

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Abstract. In the present paper strong–monotonic, monotonic and weak–monotonic reasoning is studied in the context of algorithmic language learning theory from positive as well as from positive and negative data.

Strong–monotonicity describes the requirement to *only* produce better and better generalizations when more and more data are fed to the inference device. *Monotonic learning* reflects the eventual interplay between generalization and restriction during the process of inferring a language. However, it is demanded that for any two hypotheses the one output later has to be at least as good as the previously produced one with respect to the language to be learnt. *Weak–monotonicity* is the analogue of cumulativity in learning theory.

We relate all these notions one to the other as well as to previously studied modes of identification, thereby in particular obtaining a strong hierarchy.

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1. Introduction

The process of hypothesizing a general rule from eventually incomplete data (e.g. examples, data obtained by performing experiments a.s.o.) is called inductive inference. In the philosophy of science inductive inference has attracted much attention during the last centuries. Some of the principles developed are very much alive in *algorithmic learning theory*, an emerging science starting with the seminal papers of Solomonoff (1964) and of Gold (1967). Computer scientists widely used their insight into the theory of computability to obtain a better and deeper understanding of processes performing inductive generalizations. In a beautiful paper Angluin and Smith (1987) survey the today state of the art in inductive inference.

The present paper mainly deals with formal language learning, a field in which many interesting and sometimes surprising results have been elaborated within the last decades (cf. e.g. Osherson, Stob and Weinstein (1986), Case (1988), Fulk (1990), Jain and Sharma (1990)). One of the central questions studied so far is whether or not various restrictions on the behavior of a learner do limit the learning capabilities of machines. We shall continue along this line. Before explaining what requirements we want to deal with, let us recall the general situation investigated in language learning. The learner is provided with incomplete information concerning the language to be inferred and has to produce, from time to time, a hypothesis about the phenomenon to be learnt. The information given may contain only *positive examples*, i.e., only strings from the language to be recognized, as well as both *positive and negative examples*, i.e., the learner is fed more and more strings over the underlying alphabet which are classified with respect to their containment to the unknown language. The space of hypotheses may vary from a particular set of acceptors or grammars to sets of characteristic functions. Moreover, the sequence of hypotheses is required to converge in some specified sense to a hypothesis correctly describing the object to be learnt. There are many possible requirements to the sequence of all created hypotheses. What we like to present in the sequel is an almost complete investigation of the influence of various monotonicity conditions originally introduced by Jantke (1991A) and Wiehagen (1990). The main underlying question can be posed as follows: Would it be possible to learn the unknown language in using more and more information about it and thereby producing *only* better and better generalizations? In its strongest interpretation that means we are required to infer a sequence of hypotheses describing an augmenting chain of languages, i.e., $L_i \subseteq L_j$ iff L_j is guessed later than L_i (cf. Definition 3 (A)).

This requirement finds its analogue in classical logics where an enlargement of the set of assumptions always leads to an eventually larger set of derivable theorems. However, as Jantke (1991A, 1991B) has shown, in the setting of recursive function learning, strong–monotonicity considerably restricts the inferring power. Starting from the observation that "better" has to be interpreted with respect to the goal of the learning process, i.e., with respect to the language L having to be learnt, Wiehagen (1990, 1991) proposed the following modification: Instead of demanding $L_i \subseteq L_j$ we only require $L_i \cap L \subseteq L_j \cap L$ iff L_j appears later in the sequence of created hypotheses than L_i does (cf. Definition 3 (B)). Intuitively speaking, now monotonicity means that the new hypothesis is never allowed to destroy something what a previously generated guess already *correctly* reflects.

The third version of monotonicity, which we call weak–monotonicity, is derived from

non–monotonic logics and may be interpreted as the analogue of cumulativity. Consequently, we only require $L_i \subseteq L_j$ as long as the data obtained after having produced L_i do not contradict L_i (cf. Definition 3 (C)).

As it turns out, if the learning process is performed with *positive and negative examples* then weak–monotonicity does *not restrict* the inferring power as long as the space of hypotheses is suitable chosen (cf. Theorem 6). In case one learns from positive data alone, we show that weak–monotonically working learning devices are exactly as powerful as conservatively working ones. A learning device is said to be *conservative* if and only if it only performs justified mind changes, i.e., it changes its actual hypothesis only in case it is contradicted by new data (cf. Angluin (1980A)).

All other notions of monotonicity do immediately lead to a severe limitation of the learning power, as we shall show. Finally, in order to avoid confusion it should be mentioned that there is yet another notion of monotonic learning (cf. Osherson/Stob and Weinstein (1986), Fulk (1990)) which is slightly different from ours.

The paper is structured as follows. Section 2 presents preliminaries, i.e., notations, definitions and motivations. In section 3 we exclusively deal with identification from positive data. The monotonic inference with positive and negative data is studied in section 4. Section 5 is devoted to the question whether natural families of languages may be inferred monotonically. Finally, we give a summary and present open problems. All references are listed in section 7.

2. Preliminaries

By $N = \{1, 2, 3, ...\}$ we denote the set of all natural numbers. In the sequel we assume familarity with formal language theory (cf. e.g. Bucher and Maurer (1984)). By Σ we denote any fixed finite alphabet of symbols. Let Σ^* be the free monoid over Σ. The length of a string $w \in \Sigma^*$ is denoted by $|w|$. Any subset $L \subseteq \Sigma^*$ is called a language. By $co - L$ we denote the complement of L, i.e., $co - L = \Sigma^* \setminus L$. Let L be a language and $t = s_1, s_2, s_3, ...$ a sequence of strings from Σ^* such that $range(t) = \{s_k \mid k \in N\} = L$. Then t is said to be a *text* for L or, synonymously, a positive presentation. Furthermore, let $i = (s_1, b_1), (s_2, b_2), ...$ be a sequence of elements of $\Sigma^* \times \{+, -\}$ such that $range(i) = \{s_k \mid k \in N\} = \Sigma^*$, $i^+ = \{s_k \mid (s_k, b_k) = (s_k, +), k \in N\} = L$ and $i^- = \{s_k \mid (s_k, b_k) = (s_k, -), k \in N\} = co - L$. Then we refer to i as an *informant*. If L is classified via an informant then we also say that L is represented by positive and negative data. Moreover, let t, i be a text and an informant, respectively, and let x be a number. Then t_x, i_x denote the initial segment of t and i of length x, respectively, e.g., $i_3 = (s_1, b_1), (s_2, b_2), (s_3, b_3)$. Let t be a text and let $x \in N$. Then we set $t_x^+ = \{s_i \mid i \leq x\}$. Furthermore, by i_x^+ and i_x^- we denote the sets $\{s_k \mid (s_k, +) \in i, k \leq x\}$ and $\{s_k \mid (s_k, -) \in i, k \leq x\}$, respectively.

Following Angluin (1980A) we restrict ourselves to deal exclusively with indexed families of recursive languages defined as follows:
A sequence $L_1, L_2, L_3, ...$ is said to be an *indexed family* $\mathcal{L}$ of recursive languages provided all L_i are non–empty and there is a recursive function f such that for all numbers i and all strings $w \in \Sigma^*$ we have

$$f(i,w) = \begin{cases} 1 & , \quad if \quad w \in L_i \\ 0 & , \quad otherwise. \end{cases}$$

As an example we consider the set $\mathcal{L}$ of all context–sensitive languages over Σ. Then $\mathcal{L}$ may be regarded as an indexed family of recursive languages (cf. [4]). In the sequel we often denote an indexed family and its range by the same symbol $\mathcal{L}$. What is meant will be clear from the context.

As in Gold (1967) we define an *inductive inference machine* (abbr. IIM) to be an algorithmic device which works as follows: The IIM takes as its input larger and larger initial segments of a text t (an informant i) and it either requires the next input string, or it first outputs a hypothesis, i.e., a number encoding a certain computer program, and then it requires the next input string (cf. e.g. Angluin (1980A)).

At this point we have to clarify what space of hypotheses we should choose, thereby also specifying the goal of the learning process. Wiehagen (1977) pointed out that there is a difference in what can be inferred in dependence on whether we want to synthesize in the limit grammars (i.e., procedures generating languages) or decision procedures, i.e., programs of characteristic functions. Case and Lynes (1982) investigated this phenomenon in detail. As it turns out, IIMs synthesizing grammars can be more powerful than those ones which are requested to output decision procedures. Moreover, several surprising results distinguishing language learning from inductive inference of recursive functions are originated in choosing grammars as space of hypotheses instead of characteristic functions. Since we focus in investigating the influence of monotonicity in *language learning* we decided to require the IIMs to output grammars. Furthermore, since we exclusively deal with indexed families $\mathcal{L} = (L_i)_{i \in N}$ of recursive languages we always take as space of hypotheses an enumerable family of grammars $G_1, G_2, G_3, ...$ over the terminal alphabet Σ satisfying $\mathcal{L} = \{L(G_j) \mid j \in N\}$. Moreover, we always require that membership in $L(G_i)$ is uniformly decidable for all $i \in N$ and all strings $w \in \Sigma^*$. As it turns out, it is sometimes very important to choose the space of hypotheses appropriately in order to achieve the desired learning goal. Then the IIM outputs numbers j which we interprete as G_j.

A sequence $(j_x)_{x \in N}$ of numbers is said to be convergent in the limit if and only if there is a number j such that $j_x = j$ for almost all numbers x.

Definition 1, (Gold (1967)) *Let $\mathcal{L}$ be an indexed family of languages, $L \in \mathcal{L}$, and let $(G_j)_{j \in N}$ be a space of hypotheses. An IIM M $LIM-TXT$ ($LIM-INF$)–identifies L on a text t (an informant i) iff it outputs infinitely many hypotheses and the sequence $(M(t_x))_{x \in N}$ $(M(i_x))_{x \in N}$ converges in the limit to a number j such that $L = L(G_j)$.*
Moreover, M $LIM-TXT$ ($LIM-INF$)–identifies L, iff M $LIM-TXT$ ($LIM-INF$)–identifies L on every text (informant) for L. We set:
$LIM-TXT(M) = \{L \in \mathcal{L} \mid M\ LIM-TXT-identifies\ L\}$ and define $LIM-INF(M)$ analogously.
Finally, let $LIM-TXT$ ($LIM-INF$) denote the collection of all families $\mathcal{L}$ of indexed families of recursive languages for which there is an IIM M such that $\mathcal{L} \subseteq LIM-TXT(M)$ ($\mathcal{L} \subseteq LIM-INF(M)$).

Definition 1 could be easily generalized to arbitrary families of recursively enumerable

languages (cf. [18], [8]). Nevertheless, we exclusively consider the restricted case defined above, since our motivating examples are all families of recursive languages. Note that, in general, it is not decidable whether or not M has already inferred L. In case M produces only a *single* and *correct* guess after having been fed an initial segment of a text t (or informant i) and *stops* then, we say that M *finitely* infers L on t (on i). $M\ FIN-TXT\quad(FIN-INF)$–infers L iff it finitely infers L on every text (informant). The resulting identification type is denoted by $FIN-TXT\quad(FIN-INF)$.

Next we want to formally define strong–monotonic, monotonic and weak–monotonic inference. In order to do this, first we have to explain what does it mean that an IIM *consistently* identifies a language. Consistent IIM have been introduced by Barzdin (1974). Intuitively speaking, consistency means that the IIM does correctly reflect the part of the language the IIM has already been fed with when it produces a guess.

Definition 2, Barzdin ((1974)) *An IIM M $CONS-TXT\quad(CONS-INF)$–identifies L on a text t (an informant i) iff*
(1) $M\ LIM-TXT\quad(LIM-INF)$*–identifies L on t (on i)*

(2) Whenever M on t_x (i_x) produces a hypothesis j_x then $range(t_x) \subseteq L(G_{j_x})$ ($i_x^+ \subseteq L(G_{j_x})$ and $i_x^- \subseteq co-L(G_{j_x})$.)

$M\ CONS-TXT\ (CONS-INF)$–identifies L iff $M\ \ CONS-TXT\ (CONS-INF)$–identifies L on every text t (informant i).
By $CONS-TXT(M)\quad(CONS-INF(M))$ we denote the set of all languages which M does $CONS-TXT\quad(CONS-INF)$–identify. $CONS-TXT$ and $CONS-INF$ are analogously defined as above.

Definition 3, Jantke ((1991A), Wiehagen (1991)) *An IIM M is said to identify a language L from text (informant)*
(A) strong–monotonically

(B) monotonically

(C) weak–monotonically

iff
$M\ LIM-TXT\quad(LIM-INF)$–identifies L and for any text t (informant i) of L as well as for any two consecutive hypotheses j_x, j_{x+k} which M has produced when fed t_x and t_{x+k} (i_x and i_{x+k}), for some $k \geq 1, k \in N$, the following conditions are satisfied:
(A) $L(G_{j_x}) \subseteq L(G_{j_{x+k}})$

(B) $L(G_{j_x}) \cap L \subseteq L(G_{j_{x+k}}) \cap L$

(C) if $t_{x+k} \subseteq L(G_{j_x})$ then $L(G_{j_x}) \subseteq L(G_{j_{x+k}})$ (if $i_{x+k}^+ \subseteq L(G_{j_x})$ and $i_{x+k}^- \subseteq co-L(G_{j_x})$ then $L(G_{j_x}) \subseteq L(G_{j_{x+k}})$.)

Remark: (C) in particular means that M has to work strong–monotonically as long as its guess j is consistent with the data fed to M after M has output j.

By $SMON-TXT$, $SMON-INF$, $MON-INF$, $MON-TXT$, $MON-INF$, $WMON-TXT$, $WMON-INF$ we denote the family of all thoses sets $\mathcal{L}$

of languages for which there is an IIM inferring it strong–monotonically, monotonically, and weak–monotonically from text t or informant i, respectively.

This seems to be a good place to give an example showing what inferring power our actual choice of the space of hypotheses really implies. Let $L \subseteq \Sigma^*$ be any arbitrarily fixed *infinite* context–sensitive language. By $\mathcal{L}_{fin}$ we denote the set of all finite languages over Σ. Then we set $\mathcal{L}_{finvar} = \{L \cup L_{fin} \mid L_{fin} \in \mathcal{L}_{fin}\}$. In case we would require the IIMs to output programs of characteristing functions one immediately obtains from Jantke (1991A) that, even on informant, $\mathcal{L}_{finvar}$ cannot be learnt strong–monotonically. On the other hand, in our setting $\mathcal{L}_{finvar}$ is strong–monotonically learnable, even on *text*.

The IIM M performing the inference process initially outputs a grammar G of L. Let j_x be $M's$ guess after having received t_x. If $t_x^+ \subset t_{x+1}^+$ then M tests whether the new string w belongs to $L(G_{j_x})$. In case it does, we set $j_{x+1} = j_x$. Otherwise M outputs a canonical number j_{x+1} of the grammar $G_{j_x} \cup \{\sigma \rightarrow w\}$ where σ denotes the distinguished non–terminal of G_{j_x} from which every derivation starts. Since for any L' in $\mathcal{L}_{finvar}$ there is a $L_{fin} \in \mathcal{L}_{fin}$ such that $L' = L \cup L_{fin}$ the "else" case can happen at most finitely often. Hence, M converges and by construction M does work strong–monotonically.

Finally in this section we define *conservatively* working IIMs.

Definition 4, (Angluin (1980A))
An IIM M CONSERVATIVE–TXT $(CONSERVATIVE–INF)$–identifies L on text t (on informant i), iff for every text t (informant i) the following conditions are satisfied:
(1) $L \in LIM-TXT(M)$ $(L \in LIM-INF(M))$

(2) If M on input t_x makes the guess j_x and then makes the guess $j_{x+k} \neq j_x$ at some subsequent step, then $L(G_{j_x})$ must fail to contain some string from t_{x+k} ($L(G_{j_x})$ must fail either to contain some string $w \in i_{x+k}^+$ or it generates some string $w \in i_{x+k}^-$).

CONSERVATIVE–TXT(M), CONSERVATIVE–INF(M) as well as CONSERVATIVE–TXT and CONSERVATIVE–INF are defined in an analogous manner as above.

Intuitively speaking, a conservatively working IIM performs *exclusively* justified mind changes.

Sometimes we want to combine some postulates, e.g. strong–monotonicity and consistency. That is denoted by e.g. $SMON-CONS-TXT$ or $SMON-CONS-INF$ and means that the particular IIM which has to perform the inferring process is required to work strong–monotonically *and* consistently on every text and informant, respectively. In the next section we present results concerning text identification. Note that in all what follows $\subset$ denotes proper set inclusion and $\#$ denotes incomparability of sets.

3. Monotonic Inference on Text

Our first theorem actually shows that there is a strong hierarchy between the different notions of monotonicity.

Theorem 1 $SMON-TXT \subset MON-TXT \subset WMON-TXT$

Proof. $SMON-TXT \subseteq MON-TXT$ is an immediate consequence of Definition 3. The part $MON-TXT \setminus SMON-TXT \neq \emptyset$ is proved via the following indexed family of languages:
Let $(i_1, ..., i_k)$ be any collection of pairwise distinct numbers. We set $L_{i_1,...,i_k} = (\{a\}^* \setminus \{a^{i_1}, ..., a^{i_k}\}) \cup \{b^{i_1}, ..., b^{i_k}\}$ and define $\mathcal{L}$ to be the family of all languages $L_{i_1,...,i_k}$, where $(i_1, ..., i_k) \in N^k, k \in N, k \geq 1, i_j \neq i_l$ if $j \neq l$ for all $j, l \in \{1, ..., k\}$.

First we present an IIM M inferring $\mathcal{L}$ monotonically on text. Let t be any fixed text. As long as the part of t fed to M does only contain strings over the alphabet $\{a\}$ the IIM M outputs a canonical grammar for $\{a\}^*$. In case the first string b^{l_1} appears in the text, M outputs a canonical grammar for L_{l_1}. This hypothesis remains unchanged until the next string b^{l_2} appears in the text ($l_1 \neq l_2$). Then M's new output is a canonical grammar for $L_{l_1 l_2}$, a.s.o.

Now suppose $L \in \mathcal{L}$. We have to show that M identifies L monotonically on text. Since $L \in \mathcal{L}$ there are a number k and pairwise distinct numbers $i_1, ..., i_k$ such that $L = L_{i_1,...,i_k}$. Moreover, on every text t M eventually outputs $L_{l_1,...,l_k}$ after having fed $b^{l_1}, ..., b^{l_k}$ sometimes, where $(l_1, ..., l_k)$ is a permutation of $(i_1, ..., i_k)$. Consequently, $L_{l_1,...,l_k} = L_{i_1,...,i_k}$. Hence M infers L. Due to the construction of M the IIM M performs at most $k+1$ mind changes. Therefore, we have to calculate $\{a\}^* \cap L = L_{i_1,...,i_k} \setminus \{b^{i_1}, ..., b^{i_k}\}$, $L_{l_1} \cap L = L_{i_1,...,i_k} \setminus \{b^{i_1}, ..., b^{i_k} \mid i_j \neq l_1,\ 1 \leq j \leq k\}$, $L_{l_1,...,l_k} \cap L = L_{i_1,...,i_k}$. Hence, if $j_1, ..., j_{k+1}$ are M's outputs, then we have $L(G_{j_1}) \cap L \subseteq L(G_{j_2}) \cap L \subseteq L(G_{j_{k+1}}) \cap L$, i.e., M works monotonically on text. This proves the first part.

It remains to show that there is no IIM identifying $\mathcal{L}$ strong–monotonically on text. However, we shall prove the slightly stronger result that $\mathcal{L} \notin SMON-INF$.

Suppose the converse, i.e., assume there is an IIM M such that $\mathcal{L} \in SMON-INF(M)$. Let $l_1 > 0$ be arbitrarily fixed and let i be any informant over $\Sigma = \{a, b\}$ for L_{l_1}. Then there must be an x such that M after having fed i_x outputs a canonical grammar for L_{l_1}, since otherwise M cannot infer L_{l_1}. Now, let l_2 be any fixed fixed number satisfying $l_2 > max\{|w| \mid w \in i_x\}$. Then consider i_x and $i_x \cup \{(b^{l_2}, +)\}$, two initial segments of an informant for $L_{l_1 l_2}$. Due to our choice of i_x, M first outputs j_x such that $L(G_{j_x}) = L_{l_1}$ and afterwards, on any large enough extension of $i_x \cup \{(b^{l_2}, +)\}$ that is an initial segment of an informant for $L_{l_1 l_2}$, it has to perform at least one mind change. Otherwise M cannot infer $L_{l_1 l_2}$. On the other hand, $L_{l_1} \not\subseteq L_{l_1 l_2}$. Thus M has to perform at least one mind change contradicting the strong–monotonicity condition. This completes the proof of $SMON-TXT \subset MON-TXT$.

Next we show $MON-TXT \subseteq WMON-TXT$. The problem we have to deal with is caused by the fact that any monotonically working IIM may eventually output an overgeneralized hypothesis j, i.e., $L \subset L(G_j)$, while a weakly-monotonically working IIM may not. The proof is done using the following claim:

Claim: Let $\mathcal{L}$ be any indexed family, and let M be any monotonically working IIM inferring $\mathcal{L}$. Let $L \in \mathcal{L}$ and let t be any fixed text for L. If M on t produces a guess j such that $L \subset L(G_j)$, then $L(G_j) \notin \mathcal{L}$.

Suppose the converse, i.e., there is an index x such that $M(t_x) = j, L \subset L(G_j)$ and $L(G_j) \in \mathcal{L}$. Then t_x is also an initial segment of a text for $L(G_j)$. On the other hand, t_x possesses an enlargement t_{x+k} such that $t^+_{x+k} \subseteq L$ and $j_{x+k} := M(t_{x+k})$ satisfies $L(G_{j_{x+k}}) = L$. Again, t_{x+k} is an initial segment of a text for $L(G_j)$. Consequently,

there should be an enlargement of t_{x+k}, say t_{x+k+z}, with strings from $L(G_j)$ such that $j_{x+k+z} := M(t_{x+k+z})$ and $L(G_{j_{x+k+z}}) = L(G_j)$.
Since

$$L(G_j) \supset L = L(G_{j_{x+k}}) \cap L(G_j) \subset L(G_{j_{x+k+z}}) \cap L(G_j) = L(G_{j_{x+k+z}})$$

we have found a text t for $L(G_j)$ on which M does not identify $L(G_j)$ monotonically, a contradiction. This proves the claim.

On the other hand, since we have required $\mathcal{L} = \{L(G_j) \mid j \in N\}$ for any space of hypotheses $(G_j)_{j \in N}$ the wanted inclusion follows immediately by the claim made above.

The remaining part, i.e., $WMON - TXT \setminus MON - TXT \neq \emptyset$ can be proved using the following indexed family $\mathcal{L}_{wmon}$:
Let $L_0 := \{a\}^*$ and for $i \geq 1$ set $L_i := \{a^z \mid z < i\} \cup \{b^z \mid z \geq i\}$ as well as $L_{i,j} := \{a^z \mid z < i\} \cup \{b^z \mid i \leq z < j\} \cup \{a^z \mid z \geq j\}$ for all $i, j \in N$ with $i < j$, and $i, j \geq 1$. $\mathcal{L}_{wmon}$ is defined to be the collection of all $L_i, L_{i,j}$. For the demonstration of $\mathcal{L}_{wmon} \in WMON - TXT \setminus MON - TXT$ the reader is referred to the proof of Theorem 8, claim C and D, where the slightly stronger result $\mathcal{L}_{wmon} \in WMON - TXT \setminus MON - INF$ is shown.

q.e.d.

Moreover, $SMON - TXT$ and $MON - TXT$ may already be separated on sets of regular languages. At this point it is only natural to ask what are lower and upper bounds of this hierarchy. The answer is given by the next theorem.

Theorem 2

(1) $FIN - TXT \subset SMON - TXT$ *and*

(2) $WMON - TXT = CONSERVATIVE\text{–}TXT$

Proof. By our definition, if $\mathcal{L} \in FIN - TXT(M)$, then M outputs a single guess. Hence $FIN - TXT \subseteq SMON - TXT$.

The remaining part $SMON - TXT \setminus FIN - TXT \neq \emptyset$ can be directly obtained by proving $\mathcal{L}_{finvar} \notin FIN - TXT$, where $\mathcal{L}_{finvar} = \{L \cup L_{fin} \mid L_{fin} \in \mathcal{L}_{fin}\}$, i.e., the family defined before presenting Definition 4.

Looking at assertion (2) we directly see that $CONSERVATIVE\text{–}TXT \subseteq WMON - TXT$, since a conservatively working IIM only performs justified mind changes.
Now suppose $\mathcal{L} \in WMON - TXT(M')$. We have to construct an IIM M such that $\mathcal{L} \in CONSERVATIVE\text{–}TXT(M)$. M is easily obtained from M' by simply adding a consistency test, i.e., let $j_x := M(t_x)$ and $w \in t^+_{x+k}$ for some $k = 0, 1, 2, \ldots$. Then test whether or not $w \in L(G_{j_x})$ for all w. In case it is, M repeats j_x as its hypothesis. Otherwise, it outputs $M'(t_{x+k})$.

q.e.d.

As an immediate consequence of Angluin's (1980A) Theorem 4 one directly obtains the following corollary. Nevertheless, it should be mentioned that the corollary remains valid even if one restricts itself to languages over an one letter alphabet.

Corollary 3 $WMON - TXT \subset LIM - TXT$

Next we want to deal with the combination of monotonic and consistent inference. Considering a particular family of languages, the so–called pattern languages (cf. section 5) Jantke (1991B) has shown that weak–monotonicity and consistency may be combined without any limitations concerning the inference power, while strong–monotonicity and consistency cannot. However, the negative result is mainly based on the fact that in his setting the IIMs are required to output on every input a hypothesis as well as on the particular choice of the space of hypotheses, i.e., the set of all patterns, as a careful analysis of his proof shows. We shall discuss the impact of these requirements below. Nevertheless, we have been surprised to obtain the following theorem:

Theorem 4

(1) $WMON - CONS - TXT = WMON - TXT$

(2) $MON - CONS - TXT = MON - TXT$

(3) $SMON - CONS - TXT = SMON - TXT$

Proof. We prove assertion (1). The rest can be obtained analogously. Obviously, $WMON - CONS - TXT \subseteq WMON - TXT$. For the opposite direction let $\mathcal{L}$ be any fixed indexed family of recursive languages and let M be an IIM such that $\mathcal{L} \subset WMON - TXT(M)$, i.e., $L \in WMON - TXT(M)$ for any $L \in \mathcal{L}$. We define an IIM $\hat{M}$ as follows:

Let t be any text for some language $L \in \mathcal{L}$ and let $x \in N$. When fed t_x the machine $\hat{M}$ simulates M's computation on t_y for all $y \leq x$. In case that M does not produce any hypothesis, $\hat{M}$ requests the next input and output nothing. Otherwise, let j_z be M's last guess. The machine $\hat{M}$ tests whether or not $t_x^+ \subseteq L(G_{j_z})$. In case it is $\hat{M}$ outputs j_z and requests the next input. Otherwise, the IIM $\hat{M}$ outputs nothing and requests the next input. By construction one directly obtains $L \in WMON - CONS - TXT(\hat{M})$. We omit the details.

q.e.d.

A closer look to the proof above shows that we mainly exploited the recursiveness of all languages in $\mathcal{L}$ as well as the fact that the IIMs are not required to produce on any input a hypothesis, i.e., our IIMs are not necessarily *responsive* (cf. Angluin (1980A)). Nevertheless, if one weakens the condition $\mathcal{L} = \{L(G_j) \mid j \in N\}$ to $\mathcal{L} \subseteq \{L(G_j) \mid j \in N\}$, then consistency and responsiveness may be combined with all types of monotonic inference as well as with identification in the limit ($LIM - TXT$) without any limitation concerning the inference power. On the other hand, if one insists on spaces of hypotheses containing exactly those languages the indexed family to be inferred comprises, then IIMs that work consistently, responsively and monotonically are slightly weaker than monotonically working ones. The same is true for $LIM - TXT$–identification. This result impressively shows that the choice of the space of hypotheses may not only influence the inferring power at all but may or may not allow the combination of postulates of naturalness.

4. Monotonic Inference on Informant

The first theorem in the section again establishes a hierarchy between the different types of monotonicity.

Theorem 5 $SMON-INF \subset MON-INF \subset WMON-INF$

Proof. $SMON-INF \subseteq MON-INF \subseteq WMON-INF$ is an immediate consequence of Definition 3. Moreover, in proving Theorem 1 we have shown that $MON-TXT \setminus SMON-INF \neq \emptyset$, hence we may conclude $SMON-INF \subset MON-INF$, since $MON-TXT \subseteq MON-INF$. For the demonstration of $WMON-INF \setminus MON-INF \neq \emptyset$ the reader is referred to the proof of Theorem 8, claim C and D, where we even prove $WMON-TXT \setminus MON-INF \neq \emptyset$.

q.e.d.

Moreover, while the proper set inclusions remain valid if one considers only families of regular languages, the next theorem shows the power of weak–monotonic IIM. However, the power of weak–monotonic IIMs again may be characterized as to coincide with the inferring capabilities of conservative ones.

Theorem 6

(1) $FIN-INF \subset SMON-INF$ *and*

(2) $WMON-INF = CONSERVATIVE\text{–}INF = LIM-INF$

Proof. The part $FIN-INF \subseteq SMON-INF$ is again obvious. The propper set inclusion is obtained by observing that $\mathcal{L}_{finvar} \notin FIN-INF$. On the other hand, $\mathcal{L}_{finvar} \in SMON-TXT$ as we have shown above. Hence assertion (1) follows. The second assertion can be shown in the same way as statement (2) of Theorem 2. We omit the details.

q.e.d.

Moreover, in our setting, i.e., considering only indexed families of recursive languages, we even obtain that $LIM-INF = CONS-INF$.
The next theorem shows that we again may combine monotonicity and consistency without limiting the learning power.

Theorem 7

(1) $SMON-CONS-INF = SMON-INF$

(2) $MON-CONS-INF = MON-INF$

(3) $WMON-CONS-INF = WMON-INF$

Proof. This theorem can be shown analogously as Theorem 4.

q.e.d.

Now we want to compare monotonic inference from positive data with monotonic inference from both, positive and negative data. In his seminal paper Gold (1967) has shown that the inference from positive data alone is much weaker than inference on informant. Consequently, one might well expect that monotonic inference on text is less powerful than learning monotonically on informant. However, the more interesting question is whether one can strengthen the monotonicity requirement in case one changes from positive data to positive and negative data. The opposite direction of this problem is wether the weakening of the notion of monotonicity allows the inference of sets of languages on text which may only be inferred on informant in the stronger monotonic case. Our next theorem shows that all these things are almost always different ones, i.e., in general one cannot trade monotonicity versus information presentation.

Theorem 8

(1) $MON-INF \# WMON-TXT$

(2) $MON-INF \# LIM-TXT$

(3) $SMON-INF \# MON-TXT$

(4) $FIN-INF \# SMON-TXT$

Proof. The first part we prove here is $MON-INF \setminus LIM-TXT \neq \emptyset$. We set $L_0 = \{a\}^*$, $L_k = L_0 \setminus \{a^k\}$ for $k \geq 1$, and define $\mathcal{L} = L_0, L_1, ...$ Obviously, $\mathcal{L}$ is an indexed family.

Claim A: $\mathcal{L} \in MON-INF$

The wanted IIM M works as follows: Initially, it outputs a canonical grammar j_0 for L_0. As long as some $(a^k, -)$ does not appear in the informant, the machine M outputs j_0. In case it does, M performs a mind change and outputs a canonical grammar for L_k. This hypothesis is then repeated in any subsequent step. Since $L_0 \cap L_k = L_k$ for all $k \geq 0$, the machine M works monotonically.

Claim B: $\mathcal{L} \notin LIM-TXT$

Due to Theorem 1 of Angluin(1980A) it suffices to show that there is no finite tell–tale for L_0. Suppose the converse, i.e., there is a recursively enumerable finite set T_0 satisfying:

(1) $T_0 \subset L_0$

(2) for all $j \geq 1$, if $T_0 \subseteq L_j$, then $L_j \not\subset L_0$

Let $z = max\{|w| \mid w \in T_0\}$. Then, in accordance with the definition of the languages L_k we immediately get $L_{z+1} \supseteq T_0$. Moreover, $L_{z+1} \subset L_0$, yielding a contradiction to condition (2). This proves $MON-INF \setminus LIM-TXT \neq \emptyset$.

Next we show $WMON-TXT \setminus MON-INF \neq \emptyset$ the indexed family $\mathcal{L}_{wmon}$ mentioned in the proof of Theorem 1.

Claim C: $\mathcal{L}_{wmon} \in WMON-TXT$

The wanted IIM M is informally defined as follows: Let $L \in \mathcal{L}_{wmon}$, and let t be any text for L. M initially outputs a canonical grammar for the languaguge $\{a\}^*$. In case the first string b^z appears in t the machine M outputs a grammar for L_z. This guess remains unchanged until

(α) there appears a sting b^k in the text t with $k < z$ or

(β) there appears a string a^m in the text t with $z < m$.

If (α) happens first, then M outputs a grammar for L_k, in case (β) happens first $M's$ new output is $L_{z,m}$, and finally, if (α) happens after (β) has already happened, then M produces a canonical grammar for $L_{k,m}$.
Looking at the definition of M we immediately observe that M changes its mind only in case if its current guess is inconsistent with the new data. Consequently, $\mathcal{L}_{wmon} \in WMON - TXT(M)$
Finally we have to prove the following claim:

Claim D: $\mathcal{L}_{wmon} \notin MON - INF$

Suppose the converse, i.e., assume $\mathcal{L}_{wmon} \in MON - INF(M)$ for some IIM M. Let i be any informant for $\{a\}^*$. Since $L_0 \in \mathcal{L}_{wmon}$ there must be an x such that $j_x = M(i_x)$ and $L(G_{j_x}) = L_0$. Next we successively enlarge i_x by $(b^z, +)$, where $z \geq y = max\{|w| \mid w \in i_x^+ \cup i_x^-\} + 1$. Consequently, all i_{x+k} are initial segments of an informant for L_y. Hence there must be a number k such that M on i_{x+k} outputs a grammar j_{x+k} being correct for L_y. But now we may enlarge i_{x+k} in a canonical manner to an informant i_{fool} for $L_{y,m}$ where $m = max\{|w| \mid w \in i_{x+k}^+ \cup i_{x+k}^-\} + 1$. It is easy to see that M either does not work monotonically on i_{fool} or it does not infer $L_{y,m}$. This proves the claim, and so we have proved assertion (1) and (2).
Next we prove (3) and (4). The part $MON - TXT \setminus SMON - INF \neq \emptyset$ has been shown in the proof of Theorem 1. In order to complete the proof it suffices to show that $FIN - INF \setminus MON - TXT \neq \emptyset$. Let $k \in N$, and set $L_k = \{a\}^* \setminus \{a^k\}$. We define $\mathcal{L}$ to be the family of all languages L_k. Obviously, $\mathcal{L}$ is an indexed family. The IIM M finitely inferring $\mathcal{L}$ on informant works as follows: As long as i_x does not contain a pair $(a^k, -)$, the machine M outputs nothing. In case a pair $(a^k, -)$ appears, M outputs a canonical grammar for L_k and stops.

It remains to show that $\mathcal{L} \notin MON - TXT$. Suppose it is, i.e., there is an IIM M that identifies $\mathcal{L}$ on text. Let us consider M's behavior on the following text t_{fool}. We feed $a^2, a^3, ...$ to M until it outputs a guess j_{z-1} (say on $a^2, a^3, ..., a^z$) such that $L(G_{j_{z-1}}) = L_1$. In case it does not, we are done, since M does not identify L_1 on it lexicographically ordered text. Then we input $a, a^{z+2}, a^{z+3}, ...$ until M produces a correct guess j_r for L_{z+1}. Again, there has to be an $r \geq z+2$ such that M on $a^2, a^3, ..., a^z, a, a^{z+2}, a^{z+3}, ..., a^r$ outputs j_r, since otherwise M never infers L_{z+1}. But now we may proceed as follows. We set $t_{fool} = a^2, a^3, ..., a^z, a, a^{z+2}, ..., a^r, a^{r+2}, a^{r+3}, ...$, i.e., t_{fool} is a text for L_{r+1}. Since $a^{z+1} \in L_1 \cap L_{r+1}$ but $a^{z+1} \notin L_{z+1} \cap L_{r+1}$ we get $L_1 \cap L_{r+1} \not\subseteq L_{z+1} \cap L_{r+1}$. Consequently, M does not work monotonically on t, a contradiction.

q.e.d.

Theorem 8 has the following direct consequences which are summarized in the next corollary.

Corollary 9

(1) $MON - TXT \subset MON - INF$

(2) $WMON - TXT \subset WMON - INF$

(3) $LIM - TXT \subset LIM - INF$

(4) $FIN - TXT \subset FIN - INF$

Next we ask whether assertion (2) of the latter theorem can be sharpened to $SMON - INF \setminus LIM - TXT \neq \emptyset$. The answer to this question put the hardness of strong–

monotonic inference in the right perspective.

Theorem 10 $SMON - INF \subset WMON - TXT$

Proof. It suffices to show $SMON - INF \subseteq WMON - TXT$ since by Theorem 8, assertion (1), one directly obtains $WMON - TXT \setminus SMON - INF \neq \emptyset$. Let $\mathcal{L} \in SMON - INF(M)$, for some IIM M. We have to construct an IIM M' identifying $\mathcal{L}$ weak-monotonically on text. This is done as follows: Let $L \in \mathcal{L}$ and let $t = (s_i)_{i \in N}$ be an arbitrarily fixed text for L. Then the wanted machine M' is defined as follows:

$M'(t_x) =_{df}$ "rearrange the strings contained in t_x in lexicographical order without repititions. Let $t'_k = w_1, ..., w_k$, $k \leq x$ the sequence obtained.
Compute $A_x := \{(w_1, +), ..., (w_k, +)\} \cup \{(v, -) \mid v \in \Sigma^* \setminus t_x^+, \quad |v| \leq max\{x, |w_k|\}\}$
Determine i_z by rearranging the elements of A_x in lexicographical order with respect to the first component.
(* Note that i_z is not necessarily an initial segment of an informant of L, since it may contain some $(v, -)$ such that $v \in L$. However, if x is large enough, then there is an $l \in N$ such that i_m is an initial segment of an informant for L for any $m \leq l$ *)
Compute $M(i_0)$, $M(i_1)$, ... until M outputs the first hypothesis j_r, where $r \leq z$, satisfying $i_z^+ \subseteq L(G_{j_r})$ and $i_r^- \subseteq co - L(G_{j_r})$.
(* Observe that every string $(w, +) \in i_z^+$ belongs indeed to L by our construction *)
In case j_r has been found, output it. Otherwise, output nothing, and request t_{x+1}."

It remains to show that $L \in WMON - TXT(M')$. We have to prove that

(1) M' converges to a number j such that $L = L(G_j)$, and
(2) M' works weak–monotonically

In order to show (1) remember that M works strong–monotonically on every informant for L. Consequently, M works strong–monotonically on the lexicographical ordered informant i. Hence, there is a minimal index m such that $M(i_m) = j_m$ and $L(G_{j_m}) = L$. Moreover, since t is a text for L there is an index x satisfying $i_m^+ \subseteq t_{x+k}^+$ and $i_m^- \subseteq range(i_{z+k})$ for all $k \geq 0$, where i_{z+k} is the initial segment of an informant obtained from t_{x+k} as described above. Since $L(G_{j_m}) = L$ we get:

$$i_{z+k}^+ \subseteq L(G_{j_m} \; and \; i_m^- \subseteq co - L(G_{j_m}) \; for \; every \; k \geq 0.$$

Thus, $M'(t_x) = M'(t_{x+k})$ for all $k \geq 0$, i.e., M' converges.

We proceed in showing (2). Since M works strong–monotonically on i_m we have: Whenever M on i_l, $l \leq m$ outputs a guess j_l, then $L(G_{j_l}) \subseteq L$.
Now it suffices to prove that every mind change of M' is forced by an inconsistency with the text t. Let j_r be guess produced by M', i.e., in simulating M an initial segment i_r has been found such that $M(i_r) = j_r$ and $i_z^+ \subseteq L(G_{j_r})$ as well as $i_r^- \subseteq co - L(G_{j_r})$. We distinguish the following two cases:
Case (α): i_r is an initial segment of i_m

Then $L(G_{j_r}) \subseteq L$. Consequently, if j_r is not the hypothesis M' converges to, then there has to be a k such that $i_{z+k}^+ \not\subseteq L(G_{j_r})$. But that means the text t has to contain some word $w \notin L(G_{j_r})$. Hence, this mind change is forced by an inconsistency.
Case (β): i_r is not an initial segment of i_m

Then i_r must contain a string $(v, -)$ such that $v \in L$. Since $i_r^- \subseteq co - L(G_{j_r})$ it follows that $v \notin L(G_{j_r})$. Copnsequently, in this case j_r is changed, if either a string s appears in t such that $(s, -) \in i_r^-$ or $s \in i_z^+$ and $s \notin L(G_{j_r})$. In both cases the mind change is due to inconsistency.

q.e.d.

Summarizing these results with those ones obtained previously, we get the following figure.

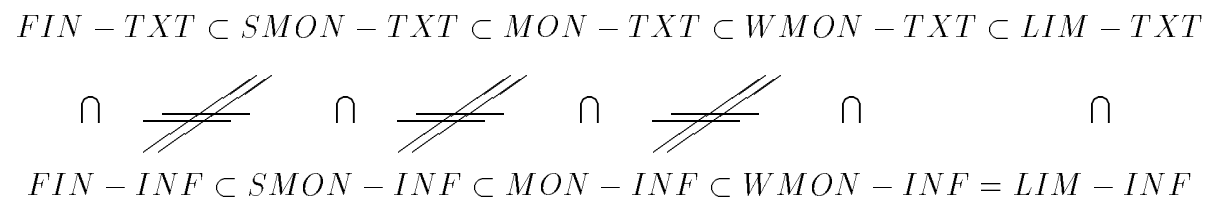

Figure 1

After having clarified the principal relations between the different notions of monotonicity we want to deal with the question what "natural" sets of languages may or may not be inferred monotonically. This is done in the next section.

5. Monotonic Lerning of Natural Families of Languages

In this section we mainly deal with the question whether sets of pattern languages may be learnt strong–monotonically or monotonically on text or on informant. Pattern languages have been introduced by Angluin (1980B), thereby proving that the whole class of pattern languages can be inferred in the limit ¿from positive data. Subsequently, Shinohara (1982) dealt with polynomial time learnability of subclasses of pattern languages. Nix (1983) outlined interesting applications of pattern inference algorithms. Recently, Kearns and Pitt (1989) as well as Ko et al. (1990) studied intensively pattern inference from positive and negative data in the PAC–learning model.

So let us define what are a pattern and a pattern language. Let Σ be any non–empty finite alphabet, and let $X = \{x_1, x_2, ...\}$ a set of variables. *Patterns* are non–empty strings of variables and constants from Σ, e.g. x_1x_2, $ax_1x_7bx_3$ are patterns. $L(p)$, the language generated by pattern p is the set of strings which can be obtained by substituting non–null strings from Σ^* for the variables of the pattern p. PAT denotes the set of all pattern languages. Finally, let DPAT be the family of all languages L for which there are patterns p_{i_1}, p_{i_2}, ..., p_{i_k} such that $L = \bigcap_{j=1}^{n} L(p_{i_j})$.

Using the techniques developed in Lange and Wiehagen (1990) one gets the following theorem:

Theorem 11

(1) $PAT \in FIN - INF$

(2) $PAT \notin FIN - TXT$

Proof. In order to prove assertion (1) let $L \in PAT$ and let i be any informant for L. The wanted IIM M is informally defined as follows:

M requests more and more inputs i_x until it finds the $k \in N$ such that the following conditions are satisfied:

(α) Every $w \in \Sigma^*$ with $|w| < k$ belongs to i_x^-.

(β) There is a $w \in \Sigma^*$ of length k that is contained in i_x^+, and all $w \in \Sigma^*$ of length k are classified in i_x.

Having found this k the machine M starts the procedure described in Lange/ Wiehagen (1990) with all strings $w \in L$ of length k and generates the uniquely determined canonical pattern p such that $L = L(p)$. Finally, it outputs p and stops.

The proof of assertion (2) can be directly obtained by observing that every text for any pattern language L is also a text for $L(x_1)$. Hence, $L(x_1)$ cannot be finitely inferred on text. We omit the details.

q.e.d.

On the other hand, Jantke (1991B) pointed out that $PAT \notin SMON - CONS - TXT$. However, this result is mainly based on the requirement that in his setting any IIM is only allowed to output patterns. If the space of hypotheses is restricted to PAT, then one can reduce the question whether at least $PAT \in SMON - TXT$ to the decidability of inclusion of pattern languages, i.e., to decide whether or not $L(p) \subseteq L(q)$ for any given patterns p and q. Unfortunately, the latter problem, first stated in Angluin (1980B), is open.

Consequently, it is only natural to ask whether an appropriate choice of the space of hypotheses may help to overcome the difficulties in designing an IIM that infers PAT strong–monotonically. Note that the next theorem is mainly based on an idea communicated to us by H.R. Beick (1991).

Theorem 12 *There is a space of hypotheses such that* $DPAT \in SMON - TXT$.

Proof. The space of hypotheses is a subset of all context–sensitive grammars obtained as follows: It is not hard to see that there is an effective procedure translating every pattern p into a context–sensitive grammar G_p such that $L(p) = L(G_p)$. Moreover, the set of all context–sensitive grammars is closed under intersection (cf. [6]), i.e., given two context–sensitive grammars G_1 and G_2 one can effectively construct a grammar G_{12} such that $L(G_{12}) = L(G_1) \cap L(G_2)$. Consequently, one can effectively construct a recursively enumerable family $(G_j)_{j \in N}$ of grammars such that $DPAT = \{L(G_j) \mid j \in N\}$.

We define an IIM M inferring $DPAT$ strong–monotonically on text as follows: Let

$L \in DPAT$ and let t be any text for L. For any $x \in N$ we set:

$M(t_x)$ = "Compute $z = min\{|w| \mid w \in t_x^+\}$ and generate all canonical patterns $p_1, ..., p_k$ with $|p_i| \leq z$. Test for all these patterns, if $t_x^+ \subseteq L(p_i)$, $i = 1, ...k$ and let I_x be the set of indices fulfilling the test.

Output a canonical grammar j_x for $\bigcap_{i \in I_x} L(p_i)$."

It remains to show that $DPAT \in SMON - TXT(M)$. Let $L \in DPAT$, i.e., there are patterns $q_1, ..., q_l$ such that $L = \bigcap_{j=1}^{l} L(q_j)$. Let $m = min\{|w| \mid w \in L\}$. Consequently, $|q_j| \leq m$ for all $j = 1, ...l$. Moreover, since t is a text for L we directly obtain that $m \leq z$ for all initial segments t_x of t and that $t_x^+ \subseteq L(q_j)$ for all $j = 1, ...l$. Thus $\{1, ..., l\} \subseteq I_x$ for every $x \in N$. Hence, every $j_x = M(t_x)$ satisfies $L(G_{j_x}) \subseteq L$. Furthermore, since $I_{x+k} \subseteq I_x$ for every $k \in N$ we additionally get $L(G_{j_x}) \subseteq L(G_{j_{x+k}}) \subseteq L$ for all $k \in N$. Consequently, M works strong–monotonically. Finally, M obviously converges since $I_0 \setminus \{1, ...l\}$ is finite. Let j_{fin} be the limit of the sequence of hypotheses. Suppose $L(G_{fin}) \subset L$. Then there is at least a string $s \in L \setminus L(G_{fin})$. On the other hand, there must be an $x \; in N$ such that $s \in t_x^+$, since t is a text for L. Due to the construction it must hold that $t_x^+ \subseteq L(p_i)$ for every $i \in I_x$, in particular $s \in L(p_i)$ for all $i \in I_x$. Hence $s \in L(G_{fin})$. Consequently, M converges to a correct grammar.

q.e.d.

On the other hand, at first glance it might seem that the decidability of $L(G) \subseteq L(G')$ may be generally helpful in proving that families of languages are monotonically learnable. However, decidability of inclusion does *not* guarantee monotonic learnability, even *on informant.*

Theorem 13 *The set of all regular languages cannot be monotonically inferred on informant.*

Proof. The proof can be done via the languages of the indexed family $\mathcal{L}_{wmon}$ introduced in the demonstration of Theorem 1. All these languages are regular. However, in an analogous manner to the proof of claim D in the demonstration of Theorem 8 one can show that there is no IIM M inferring all the regular languages monotonically on informant. We omit the details.

q.e.d.

Let us finish this section with the remark that several problems remained open. We shall discuss them in the next section.

6. Conclusions and Open Problems

We have studied the power of strong–monotonic, monotonic and weak–monotonic IIMs in the setting of algorithmic language learning. All these notions have been related one to the other and two new hierarchies could been proven. Although all these notions seem to be quite natural ones, in general, strong–monotonicity and monotonicity lead to a severe restriction of the inference power. This actually shows that learning processes have to be performed, at least to some extent, non–monotonically or at least weak–monotonincally. Moreover, as our results show, if one deals with monotonically working IIMs the choice of the space of hypotheses is of great influence to what actually

can be inferred. If that space is appropriately chosen, then weak–monotonic inference can be characterized to be as powerful as conservative learning.

However, several problems remained open. First, it would be very interesting to know whether or not the requirement that the range of the indexed family to be learnt has to coincide with the range of the space of hypotheses is essential for proving $MON-TXT \subseteq WMON-TXT$. Moreover, looking at the discussion we presented after having proven Theorem 4, it seems sometimes very natural to enlarge the collection of allowed spaces of hypotheses. This impression may be intensified if one looks to the following result. There is a space of hypotheses such that $PAT \in SMON-TXT$. In proving the latter theorem we have chosen the range of the used space of hypotheses considerably larger than the collection of all pattern languages. Moreover, it seems to be challenging to combine monotonic language learning with other postulates of naturalness, e.g., with iterative inference introduced by Wiehagen (1976). It should be mentioned that iteratively working IIMs have to be responsive.

Finally, it would be desirable to prove sufficient and necessary conditions for all the types of monotonic inference introduced above. We shall present results along this line of research in a subsequent paper.

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