# Complexity and Cryptography 

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Lecture 11: Classical Two-Way Cryptosystems


## Before we start

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There are two kinds of cryptography in this world： cryptography that will stop your kid sister from reading your files，and cryptography that will stop major governments from reading your files．

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We shall mainly deal with the latter．

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Install and use GnuPG. It is available from
http://www.gnupg.org/

## Two famous opinions

"Ceux qui se vantent de lire les lettres chifrées sont de plus grands charlatans que ceux qui se vanteraient d'entendre une langue qu'ils n'ont point apprise."

Voltaire (Dictionnaire philosophique, 1769)
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Part of the course is devoted to finding out who of those famous thinkers is closer to the truth.

## Cryptology I

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Cryptology can be diveded into two major parts, i.e., cryptography and cryptanalysis.

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Cryptography is the science or art of secret writing while cryptanalysis is its natural counterpart, that is, the art of reading secret messages. A classic goal of cryptography is privacy: two or more parties wish to communicate in a way such that an adversary knows nothing about what was communicated.

## The Basic Model I



Figure 1: The Basic Model

## The Basic Model II

The message we want send is called plaintext. However, only the intended recipients should be able to read and to understand the message sent. Thus, messages are sent in disguised form, and the disguised message is called the ciphertext.

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We are confronted with contradictory requirements. Encryption and decryption should be "easy;" i.e., they should be computable using a reasonable amount of space and time. On the other hand, decryption should be "hard;" i.e., the adversary should either not be able to decipher the message eavesdropped in principal or it should be computationally infeasible for her to do so.

## Caesar's System I

We exemplify this basic model using a cryptosystem invented by Julius Caesar.

$$
\begin{aligned}
& \text { A BCDEFGHIJKLMNOPQRSTUVWXYZb } \\
& \text { YZbABCDEFGHI JKLMNOPQRSTUVWX }
\end{aligned}
$$

Figure 2: The Caesar system

A plaintext is easily encrypted by replacing each letter in it by the corresponding letter displayed in the second row of the above Figure, i.e., $A$ is replaced by $\mathrm{Y}, \mathrm{B}$ is replaced by Z and so on. For example, WHY is encrypted into TEV.
The secret key is just the table above. Decryption is just the opposite.

## Cryptanalysis I

This cryptosystem fulfills the first two requirements established above, i.e., encryption and deciphering are easy to compute provided the table is known.

## Question

Does it also fulfill the 3rd requirement?
What can be said about the complexity of cryptanalysis in this case?

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Answering these questions requires some care. We have two distinguish two cases.
Case 1. The cryptosystem itself is unknown.
Case 2. The principal cryptosystem is known but the actual key is is unknown.

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(1) If a cryptosystem is hard to break in Case 2, it is even harder to break in Case 1. Thus, we are on the safer side when assuming Case 2.
(2) The experience gained shows that the principal structure of a cryptosystem cannot be kept secret for a long time. Thus, we are again safer when assuming Case 2.

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(2) The experience gained shows that the principal structure of a cryptosystem cannot be kept secret for a long time. Thus, we are again safer when assuming Case 2.
So, we follow Kerckhoffs' (1883) principle: A cryptosystem is secure, if one, knowing the cryptosystem and the algorithms used, cannot decipher the cryptotext and obtain the plaintext unless the key used is known.

## Cryptanalysis III

We generally distinguish the following sources of information available to an eavesdropper: Her task is to decipher the whole messages or at least part of them.

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Case 2.3. Ciphertext obtained from plaintext chosen by the adversary.
In this scenario, the adversary has been able to force the sender to encrypt some plaintext carefully chosen by herself.

## Cryptanalysis IV

As we shall see later, the third scenario is also well conceivable, and part of the design of a cryptosystem has to be devoted to avoid such attacks to a large extent. Now, let us attack Caesar's system.

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So, in Case 2.3 the adversary has no difficulties at all. The same applies mutatis mutandis to Case 2.2.
There are only 27 cyclical shifts. Thus, even in Case 2.1 the adversary has no principal difficulty to decipher the message received. Trying all possibilities is feasible and leads to successful encryption.

## Cryptanalysis V

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Let us again take our alphabet $\mathcal{A}$ and as the set of all possible keys we consider all permutations of $\mathcal{A}$. This would be the most general version of the Cesar system.
Thus, we have 27 ! many keys, and since $27!\leqslant 8 \cdot 10^{27}$ just trying them all is not feasible. Even if we could test $10^{9}$ many permutations per second, this exaustive testing would take roughly $10^{11}$ years.

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So, at first glance, everything looks fine. Unfortunately, there is a "but," and in this case it sounds "but there is frequency analysis."

## Cryptanalysis VI

The background of frequency analysis is the observation that letters appear with different frequencies in natural language. For example, in German we have the following picture:

| E | $18.46 \%$ | R | $7.14 \%$ | T | $5.22 \%$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| N | $11.42 \%$ | S | $7.04 \%$ | U | $5.01 \%$ |
| I | $8.02 \%$ | A | $5.38 \%$ | D | $4.94 \%$ |

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Note that there is no absolute table for the relative frequencies of letters, since they vary in dependence on the subjects. For instance, if we compute frequencies in stock market reports and book of tales, then you get different values. Nevertheless, in German texts the letters E and N always have the highest frequency.

## Cryptanalysis VII

Now, the idea of frequency analysis is to compute the frequencies in the ciphertext and to try a mapping with respect to the table displayed above. It works very often quite well.

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So far, we have considered cryptosystems that enciphered all plaintext message units using one and the same rule. Such cryptosystems are referred to as monoalphabetic systems. In contrast, in the following we study cryptosystems working as follows: The first plaintext message unit is enciphered using Rule 1, the second plaintext message unit is enciphered using Rule $2, \ldots$, the $k$ th plaintext message unit is enciphered applying Rule $k$. In case the plaintext contains more than $k$ plaintext message units, one applies the rules modulo $k$. Such systems are called polyalphabetic.

Keyword: MAGIC Message: CRYPTOLOGY


Ciphertext:

## Keyword: MAGIC Message: CRYPTOLOGY



Ciphertext: O

## Keyword: MAGIC Message: CRYPTOLOGY



Ciphertext: OR

## Keyword: MAGIC Message: CRYPTOLOGY



Ciphertext: ORE

## The Vigenère System I

Keyword: MAGIC Message: CRYPTOLOGY


Ciphertext: OREX

## The Vigenère System I

Keyword: MAGIC Message: CRYPTOLOGY


Ciphertext: OREXV

## The Vigenère System I

## Keyword: MAGIC Message: CRYPTOLOGY

|  |  | C |  | E |  |  | H | I |  |  |  |  |  | O |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W |  |  | Z |  |
| C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B |
| D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C |
| E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C |  |
| F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B |  | D |  |
| G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C |  | E |  |
| H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E |  |  |
| I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |
| J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  |
| K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  |  |
|  | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  |  |  |
| M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  | J | K |  |
| N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K |  |  |
| O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  | J | K |  |  |  |
| P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I |  | K | L | M | N |  |
| Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |  |
| R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  | J | K | L | M |  | O |  |  |
|  | T | U | V | W | X | Y | Z | A | B | C | D | E |  |  |  |  | J | K | L | M | N | O |  |  |  |
|  | U | V | W | X | Y | Z | A | B | C | D | E |  | G | H | I |  | K | L | M | N | O |  | Q | R |  |
| U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S |  |
| V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |  |
| W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U |  |
| X | Y | Z | A | B | C | D | E | F | G | H | I | J | K |  | M |  | O |  | Q | R | S |  |  |  |  |
|  | Z | A | B | C | D | E | F | G | H | I | J | K | L |  | N | O |  | Q |  | S | T |  |  |  |  |
|  | A |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Ciphertext: OREXVA

## The Vigenère System I

Keyword: MAGIC Message: CRYPTOLOGY


Ciphertext: OREXVAL

## The Vigenère System I

Keyword: MAGIC Message: CRYPTOLOGY


Ciphertext: OREXVALU

## Keyword: MAGIC Message: CRYPTOLOGY



Ciphertext: OREXVALUO

## The Vigenère System I

Keyword: MAGIC Message: CRYPTOLOGY

|  |  | C |  | E |  |  | H | I |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W |  |  |  |  |
| C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B |
| D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C |
| E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D |
| F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D |  |
| G | H | I | J | K | L | M | N | O | P | Q | R | S |  | U | V | W | X |  | Z | A | B | C | D | E |  |
| H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F |  |
| I | J | K | L | M | N | O | P | Q |  | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G |  |
| J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  |
| K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  |  |
|  | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  |  |  |
| M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  | J | K |  |
| N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I |  | K |  |  |
| O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  | J | K |  |  |  |
| P | Q | R | S | T | U | $V$ | W | X | Y | Z | A | B | C | D | E | F | G | H | I |  | K | L | M | N | O |
| Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |  |
| R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I |  | K | L | M |  | O |  |  |
|  | T | U | V | W | X | Y | Z | A | B | C | D | E |  |  | H |  | J | K | L | M | N | O |  |  |  |
|  | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |  | Q |  |  |
| U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S |  |
| V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |  |
| W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |  |  |
| X | Y | Z | A | B | C | D | E | F | G | H | I | J |  |  | M | N | O | P | Q | R | S |  |  |  |  |
|  | Z | A | B | C | D | E | F | G | H | I | J | K |  |  | N | O | P | Q |  |  | T |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Ciphertext: OREXVALUOA

## The Vigenère System II

Vigenère published his system in 1585 and it took roughly 300 years before it was broken. It is the periodicity of the repeating key which leads to the weaknesses in this method and its vulnerabilities to cryptanalysis. Wilhelm Kasiski published in 1863 his book Die Geheimschriften und die Dechiffrir-Kunst containing his famous algorithm.


Blaise de Vigenère (1523-1596)

We therefore continue here with Kasiski's algorithm.

## Kasiski's Algorithm I

Step 1. Search all words $v_{0}, \ldots, v_{\ell}$ in the ciphertext that appear at least twice in the ciphertext, i.e., search all $v_{i}$ such that the ciphertext can be presented as $w_{i} v_{i} q_{i} v_{i} r_{i}$, where $w_{i}, q_{i}, r_{i}$ are also words over the cipher alphabet.

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Step 2. For each $v_{i}$ found, $i=0, \ldots \ell$, compute all divisors of $\left|v_{i} q_{i}\right|$.

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Step 2. For each $v_{i}$ found, $i=0, \ldots \ell$, compute all divisors of $\left|v_{i} q_{i}\right|$.
Step 3. Order the divisors found in Step 2 by their frequency.
Starting with the most frequent one try for each divisor a monoalphabetic attack until a "meaningful" plaintext has been discovered.

## Kasiski's Algorithm II

## Lemma 1 (Kasiski)

Let d be the length of the key word used. Then, for every key word of length d , the corresponding Vigenère Substitution can be decomposed into d monoalphabetic substitutions.

Proof. Let $w=s_{0} \ldots s_{d-1}$ be any key word of length $d$, and let $k_{0} k_{1} \ldots k_{m}$ be the plaintext to be enciphered. We write the plaintext in blocks of length $d$ below the key word:

$$
\begin{array}{llll}
\mathrm{s}_{0} & \mathrm{~s}_{1} & \ldots & \mathrm{~s}_{\mathrm{d}-1} \\
\mathrm{k}_{0} & \mathrm{k}_{1} & \ldots & \mathrm{k}_{\mathrm{d}-1} \\
\mathrm{k}_{\mathrm{d}} & \mathrm{k}_{\mathrm{d}+1} & \ldots & \mathrm{k}_{2 \mathrm{~d}-1} \\
\mathrm{k}_{2 \mathrm{~d}} & \mathrm{k}_{2 \mathrm{~d}+1} & \ldots & \mathrm{k}_{3 \mathrm{~d}-1} \\
\cdot & & & \\
\cdot & & & \\
\cdot & & & \\
\mathrm{k}_{\ell \mathrm{d}} & \mathrm{k}_{\ell \mathrm{d}+1} & \ldots & \mathrm{k}_{\mathrm{m}} .
\end{array}
$$

## Kasiski's Algorithm III

Hence, all plaintext message units in column $i \in\{0, \ldots, d-1\}$ are enciphered by the same monoalphabetic substitution defined by letter $s_{i}$ of the key word. More precisely, the first letter of the alphabet $\mathcal{A}$ is mapped to $s_{i}$; thus canonically defining a shift operation for the remaining letters.

## Kasiski's Algorithm III

Hence, all plaintext message units in column $i \in\{0, \ldots, d-1\}$ are enciphered by the same monoalphabetic substitution defined by letter $s_{i}$ of the key word. More precisely, the first letter of the alphabet $\mathcal{A}$ is mapped to $s_{i}$; thus canonically defining a shift operation for the remaining letters.

So, we should give it a try. The example is from Salomaa (1990). The following ciphertext has been eavesdropped:

## Kasiski's Algorithm IV

AVXZHHCSBZHALVXHFMVTLHIGH KALBRVIMOFHDKTASKVBMOSLAC GLGMOSTPFULQHTSLTCKLVNTWW HBWMS X S GAVHMLFRVI TYSMOI LH P E L HHL L I L F B L BVLPHAVWYMTUR ABABKVXHHBUGTBBTAVXHFMVTL HI GHPNP ZWPBZPGGVHWPGVBGLL RALFXAVXTCLAQHTAHUABZHTRS B U P NP ZW P B ZHGTB B TPGMVVTCSM VCLTOESOLACOLKBAVMVCYLKLA CGLGBMHALGMVJXPGHUZRHABZS KHPELHBUMFLHTSPHEKBAVTJCN WZXVTLACGLGHUHHWHALBMOSKV C F J OGUCMISALOMLRIYCI LFEF I GS S L ZWMP GOLFRZATSZGL J XYP X Z H B UUR DWMOH A LVXHFMV T L H I G H

What does it mean?

AVXZHHCSBZHALVXHFMVTLHIGH K A L B R V I MOFHDK TASKVBMOSLAC GLGMOS T P F ULQHTSLTCKLVNTWW HBWMSX S GAVHMLFRVITYSMOILH P E L HHL L I L F B L B V L P HAVWYMTUR A B A B K V X HHBUGTBBTAVXHFMVT L H I GHPNPZWPBZPGGVHWPGVBGLL RALFXAVXTCLAQHTAHUABZHTRS B UP NP ZW P B ZHGTB B T P GMVVTCSM VCLTOESOLACOLKBAVMVCYLKLA CGLGBMHALGMV J X P GHUZRHABZS KHP ELHBUMFLHTSPHEKBAVTJCN WZXVTLACGLGHUHHWHALBMOSKV C F J OGUCMISALOMLRIYCILFEFI GS S L ZWM P GOL FRZATSZGLJXYPX Z H B U U R DWMOHALVXHFMVT LHIGH

AVXZHHCSBZHALVXHFMVTLHIGH K A L B R V I MOFHDK TASKVBMOSLAC GLGMOS T P F ULQHTSLTCKLVNTWW HBWMSX S GAVHMLFRVITYSMOILH P E L HHL L I L F B L B V L P HAVWYMTUR A B A B K V X HHBUGTBBTAVXHFMVT L H I GHPNPZWPBZPGGVHWPGVBGLL RALFXAVXTCLAQHTAHUABZHTRS B UP NP ZW P B ZHGTB B T P GMVVTCSM VCLTOESOLACOLKBAVMVCYLKLA CGLGBMHALGMV J X P GHUZRHABZS KHP ELHBUMFLHTSPHEKBAVT JCN WZ X V T L A C GLGHUHHWHA L B MOS K V C F J OGUCMISALOMLR I YCILFEF I GS S L ZWM P GOL FRZATSZGLJXYPX Z H B UUR DWMOHALVXHFMVTLHIGH

Step 1 gives $v_{0}=$ HALVXHFMVTLHIGH having $\left|v_{0} \mathrm{q}_{\mathbf{0}}\right|=\mathbf{3 7 5}$

AVXZHHCSBZHALVXHFMVTLHIGH K A L B R V I MOFHDKTASKVBMOSLAC GLGMOSTP FULQHTSLTCKLVNTWW HBWMS X S G A VHML F R V I T Y S MOI L H P E L HHL L I L F B L B V L P HAVWYMTUR A B A B K V X HHBUGTB B TAVXHFMVT L H I GHPNPZWPBZPGGVHWPGVBGLL RALFXAVXTCLAQHTAHUABZHTRS B UP NP ZW P B ZHGTB B T P GMVVTCSM VCLTOESOLACOLKBAVMVCYLKLA CGLGBMHALGMV J X P GHUZRHABZS KHPELHBUMFLHTSPHEKBAVTJCN WZXVTLACGLGHUHHWHALBMOSKV C F J OGUCMISALOMLRIYCILFEFI GSSLZWMPGOLFRZATSZGLJXYPX ZHBUURDWMOHALVXHFMVTLHIGH
Step 1 gives $v_{0}=$ HALVXHFMVTLHIGH having $\left|\boldsymbol{v}_{\mathbf{0}} \mathbf{q}_{\mathbf{0}}\right|=\mathbf{3 7 5}$ and

AVXZHHCSBZHALVXHFMVTLHIGH K A L B R V I MOFHDKTASKVBMOSLAC GLGMOSTP FULQHTSLTCKLVNTWW HBWMSXSGAVHMLFRVITYSMOILH P E L HHL L I L F B L B VLPHAVWYMTUR A B A B KV X HHBUGTBBTAVXHFMVT L H I GHPNPZWPBZPGGVHWPGVBGLL RALFXAVXTCLAQHTAHUABZHTRS B UP NP ZW P B ZHGTB B T P GMVVTCSM VCLTOESOLACOLKBAVMVCYLKLA CGLGBMHALGMV J X P GHUZRHABZS KHPELHBUMFLHTSPHEKBAVTJCN WZXVTLACGLGHUHHWHALBMOSKV C F J OGUCMISALOMLRIYCILFEFI GS S L ZWMPGOLFRZATSZGLJXYPX ZHBUURDWMOHALVXHFMVTLHIGH
Step 1 gives $v_{0}=$ HALVXHFMVTLHIGH having $\left|v_{0} \mathrm{q}_{0}\right|=\mathbf{3 7 5}$ and $v_{1}=$ VXHFMVTLHIGH having $\left|v_{1} \mathrm{q}_{1,0}\right|=129$ (first and second) and $\left|v_{1} \mathrm{q}_{1,1}\right|=246$ (second and third).

AVXZHHCSBZHALVXHFMVTLHIGH K A L B R V I MOFHDK T A S KVBMOSLAC GLGMOS T P F ULQHTSLTCKLVNTWW HBWMSXSGAVHMLFRVITYSMOILH P E L HHL L I L F B L B V L P HAVWYMTUR A B A B KV X HHBUGTBBTAVXHFMVT L H I GHPNPZWPBZPGGVHWPGVBGLL RALFXAVXTCLAQHTAHUABZHTRS B U P N P ZW P B ZHGTBBTPGMVVTCSM VCLTOESOLACOLKBAVMVCYLKLA CGLGBMHALGMV J X P GHUZRHABZ S KHPELHBUMFLHTSPHEKBAVTJCN WZXVTLACGLGHUHHWHALBMOSKV C F J OGUCMISALOMLRIYCILFEFI GS S L ZWMP GOLFRZATSZGLJXYPX Z H B U UR DWMOHALVXHFMVTLHIGH In Step 1 we find $\left|v_{0} q_{0}\right|=375,\left|v_{1} q_{1,0}\right|=129,\left|v_{1} q_{1,1}\right|=246$,

AVXZHHCSBZHALVXHFMVTLHIGH K A L B R V I MOFHDK TASKVBMOSLAC GLGMOS T P F ULQHTSLTCKLVNTWW HBWMSXSGAVHMLFRVITYSMOILH P E L HHL L I L F B L B VLPHAVWYMTUR A B A B KV X HHBUGTBBTAVXHFMVT L H I GHPNPZWPBZPGGVHWPGVBGLL RALFXAVXTCLAQHTAHUABZHTRS B U P N P ZW P B ZHGTBBTPGMVVTCSM VCLTOESOLACOLKBAVMVCYLKLA CGLGBMHALGMV J X P GHUZRHABZ S KHP ELHBUMFLHTSPHEKBAVTJCN WZXVTLACGLGHUHHWHALBMOSKV CF J OGUCMISALOMLRIYCILFEFI GS S L ZWMP GOLFRZATSZGLJXYPX ZHBUUR DWMOHALVXHFMVTLHIGH In Step 1 we find $\left|v_{0} \mathrm{q}_{0}\right|=375,\left|v_{1} \mathrm{q}_{1,0}\right|=129,\left|v_{1} \mathrm{q}_{1,1}\right|=246$, and VXH (in the 6th row, with distance 12)

AVXZHHCSBZHALVXHFMVTLHIGH K A L B R V I MOFHDK TASKVBMOSLAC GLGMOS T P F ULQHTSLTCKLVNTWW HBWMSXSGAVHMLFRVITYSMOILH P E L HHL L I L F B L B VLPHAVWYMTUR A B A B KV X HHBUGTBBTAVXHFMVT L H I GHPNPZWPBZPGGVHWPGVBGLL RALFXAVXTCLAQHTAHUABZHTRS B U P N P ZW P B ZHGTBBTPGMVVTCSM VCLTOESOLACOLKBAVMVCYLKLA CGLGBMHALGMV J X P GHUZRHABZ S KHP ELHBUMFLHTSPHEKBAVTJCN WZXVTLACGLGHUHHWHALBMOSKV CF J OGUCMISALOMLRIYCILFEFI GS S L ZWMPGOLFRZATSZGLJXYPX Z H B UUR DWMOHALVXHFMVTLHIGH In Step 1 we find $\left|v_{0} \mathrm{q}_{0}\right|=375,\left|v_{1} \mathrm{q}_{1,0}\right|=129,\left|v_{1} \mathrm{q}_{1,1}\right|=246$, and VXH (in the 6th row, with distance 12) and AVX with distances 141, 39, VX gives also 180,

AVXZHHCSBZHALVXHFMVTLHIGH K A L B R V I MOFHDKTASKVBMOSLAC GLGMOS T P F ULQHTSLTCKLVNTWW HBWMSXSGAVHMLFRVITYSMOILH P E L HHL L I L F B L B VLPHAVWYMTUR A B A B KV X HHBUGTBBTAVXHFMVT L H I GHPNPZWPBZPGGVHWPGVBGLL RALFXAVXTCLAQHTAHUABZHTRS B U P N P ZW P B ZHGTBBTPGMVVTCSM VCLTOESOLACOLKBAVMVCYLKLA CGLGBMHALGMV J X P GHUZRHABZ S KHPELHBUMFLHTSPHEKBAVTJCN WZXVTLACGLGHUHHWHALBMOSKV CF J OGUCMI SALOMLRI YCI LFEF I GS S L ZWMP GOLFRZATSZGLJXYPX Z H B UUR DWMOHALVXHFMVTLHIGH In Step 1 we find $\left|v_{0} \mathrm{q}_{0}\right|=375,\left|v_{1} \mathrm{q}_{1,0}\right|=129,\left|v_{1} \mathrm{q}_{1,1}\right|=246$, and VXH (in the 6th row, with distance 12) and AVX with distances 141, 39, VX gives also 180, and HAL with distances 246, 60, 69.

## Computing Divisors

for 375 : $1,3,5,25,125,15,75,375$,

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for 246: $1,2,3,41,6,82,123,246$,

## Computing Divisors

for 375 : $1,3,5,25,125,15,75,375$,
for 129: $1,3,43,129$,
for 246: 1, 2, 3, 41, 6, 82, 123, 246,
for 180: $1,2,3,4,6,5,10,15,20,45,12,36,180$,

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for 246: 1, 2, 3, 41, 6, 82, 123, 246,
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for 60: nothing new, because 60 divides 180,

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for 60: nothing new, because 60 divides 180,
for 39: $1,3,13,39$,
for 69: $1,3,23,69$, and
for 12: nothing new.

## Kasiski's Algorithm V

Thus, 3 is the most frequent divisor found, since it divides all distances.
Moreover, since several words have been pretty long, it is highly improbable that this is just by chance.

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Consequently, we conjecture the key word length to be 3. In order to perform the monoalphabetical attacks, we rewrite the ciphertext in three columns as described above and obtain:

| $s_{0} s_{1} \mathrm{~s}_{2}$ | $\mathrm{s}_{0} \mathrm{~s}_{1} \mathrm{~s}_{2}$ | $\mathrm{s}_{0} \mathrm{~S}_{1} \mathrm{~S}_{2}$ | $\mathrm{s}_{0} \mathrm{~s}_{1} \mathrm{~s}_{2}$ | $\mathrm{s}_{0} \mathrm{~s}_{1} \mathrm{~s}_{2}$ | $\mathrm{s}_{0} \mathrm{~s}_{1} \mathrm{~s}_{2}$ | $\mathrm{S}_{0} \mathrm{~S}_{1} \mathrm{~S}_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A V X | L Q H | Y M T | A V X | A V M | W Z X | L F R |
| Z H H | T S L | U R A | T C L | V C Y | V T L | Z A T |
| C S B | T C K | B A B | A Q H | L K L | A C G | S Z G |
| Z H A | L V N | K V X | T A H | A C G | L G H | L J X |
| L V X | T W W | H H B | U A B | L G B | U H H | Y P X |
| H F M | H B W | U G T | Z H T | M H A | W H A | Z H B |
| V T L | M S X | B B T | R S B | L G M | L B M | U U R |
| H I G | S G A | A V X | U P N | V J X | O S K | D W M |
| H K A | V H M | H F M | P Z W | P G H | V C F | O H A |
| L B R | L F R | V T L | P B Z | U Z R | J O G | L V X |
| V I M | V I T | H I G | H G T | H A B | U C M | H F M |
| O F H | Y S M | H P N | B B T | Z S K | I S A | V T L |
| D K T | O I L | P Z W | P G M | H P E | L O M | H I G |
| A S K | H P E | P B Z | V V T | L H B | L R I | H |
| V B M | L H H | P G G | C S M | U M F | Y C I |  |
| O S L | L L I | V H W | V C L | L H T | L F E |  |
| A C G | L F B | P G V | T O E | S P H | F I G |  |
| L G M | L B V | B G L | S O L | E K B | S S L |  |
| O S T | L P H | L R A | A C O | A V T | Z W M |  |
| P F U | A V W | L F X | L K B | J C N | P G O |  |

## Counting Letters

For each column $\mathbf{s}_{\mathbf{0}}, \mathbf{s}_{\mathbf{1}}, \mathbf{s}_{\mathbf{2}}$ counting yields:

| Letter | $\mathbf{s}_{\mathbf{0}}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ | Letter | $\mathbf{s}_{\mathbf{0}}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 12 | 5 | 9 | N | 0 | 0 | 4 |
| B | 4 | 9 | 12 | O | 6 | 4 | 2 |
| C | 2 | 11 | 0 | P | 10 | 7 | 0 |
| D | 2 | 0 | 0 | Q | 0 | 2 | 0 |
| E | 1 | 0 | 4 | R | 1 | 3 | 5 |
| F | 1 | 10 | 2 | S | 5 | 13 | 0 |
| G | 0 | 13 | 10 | T | 6 | 4 | 13 |
| H | 15 | 14 | 11 | U | 9 | 1 | 1 |
| I | 1 | 7 | 3 | V | 14 | 11 | 2 |
| J | 2 | 2 | 0 | W | 2 | 3 | 6 |
| K | 1 | 5 | 4 | X | 0 | 1 | 12 |
| L | 27 | 1 | 13 | Y | 4 | 0 | 1 |
| M | 2 | 2 | 17 | Z | 7 | 5 | 2 |

## Statistical Information for English

| E | $12.31 \%$ | O | $7.94 \%$ | S | $6.59 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | $9.59 \%$ | N | $7.19 \%$ | R | $6.03 \%$ |
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There are two such triples: TUV and YZA

## Looking at TUV and YZA

| Letter | $\mathbf{s}_{\mathbf{0}}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ | Letter | $\mathbf{s}_{\mathbf{0}}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 12 | 5 | 9 | N | 0 | 0 | 4 |
| B | 4 | 9 | 12 | O | 6 | 4 | 2 |
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| F | 1 | 10 | 2 | S | 5 | 13 | 0 |
| G | 0 | 13 | 10 | T | 6 | 4 | 13 |
| H | 15 | 14 | 11 | U | 9 | 1 | 1 |
| I | 1 | 7 | 3 | V | 14 | 11 | 2 |
| J | 2 | 2 | 0 | W | 2 | 3 | 6 |
| K | 1 | 5 | 4 | X | 0 | 1 | 12 |
| L | 27 | 1 | 13 | Y | 4 | 0 | 1 |
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Assuming $\mathbf{R} \rightarrow \mathbf{T}, \mathbf{S} \rightarrow \mathbf{U}$, and $\mathbf{T} \rightarrow \mathbf{V}$ results in conjecturing a monoalphabetic right shift by two positions, i.e.,

## Relooking at TUV and YZA

Assuming $\mathbf{R} \rightarrow \mathbf{T}, \mathbf{S} \rightarrow \mathbf{U}$, and $\mathbf{T} \rightarrow \mathbf{V}$ results in conjecturing a monoalphabetic right shift by two positions, i.e.,

> A BCDEFGHI JKLMNOPQRSTUVWXYZ
> CDEFGHI JKLMNOPQRSTUVWXYZAB

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$$
\begin{aligned}
& \text { A BCDEFGHI JKLMNOPQRSTUVWXYZ } \\
& \text { CDEFGHI JKLMNOPQRSTUVWXYZAB }
\end{aligned}
$$

Thus, $\mathrm{Y}, \mathrm{Z}$, and A would be the image of $\mathrm{W}, \mathrm{X}$, and Y , respectively. Consequently, the letters $\mathrm{W}, \mathrm{X}$, and Y must appear 4,7 , and 12 times, respectively, in the plaintext. This seems highly unlikely.

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Therefore, we favor $\mathbf{R} \rightarrow \mathbf{Y}, \mathbf{S} \rightarrow \mathbf{Z}$, and $\mathbf{T} \rightarrow \boldsymbol{A}$ resulting in:

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$$
\begin{aligned}
& \text { A BCDEFGHIJKLMNOPQRSTUVWXYZ } \\
& \text { HI J KLMNOPQRSTUVWXYZABCDEFG }
\end{aligned}
$$

We find ABC and FGH (possibly ZAB and GHI, too; but they are less probable). Using similar arguments as above, ABC is less probable than FGH. Thus, we continue working with

## Looking at the 2nd Column

We find ABC and FGH (possibly ZAB and GHI, too; but they are less probable). Using similar arguments as above, ABC is less probable than FGH. Thus, we continue working with

A BCDEFGH I J KLMNOPQRSTUVWXYZ<br>OPQRSTUVWXYZABCDEFGHIJKLMN

## Looking at the 3rd Column

Here, KLM and FGH are possible candidates. First, we favor KLM; thus obtaining:

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$$
\begin{aligned}
& \text { A B C D EF GHI J KLMNOPQRSTUVWXYZ } \\
& \text { T UVWXYZABCDEFGHI JKLMNOPQRS }
\end{aligned}
$$

Our three conjectures

$$
\begin{aligned}
& \mathbf{t}_{0}(x)=x+7 \bmod 26, \\
& \mathbf{t}_{1}(x)=x+14 \bmod 26, \\
& \mathbf{t}_{2}(x)=x+19 \bmod 26,
\end{aligned}
$$

i.e.,

## Putting it all Together

Our three conjectures

$$
\begin{aligned}
\mathfrak{t}_{0}(x) & =x+7 \bmod 26 \\
\mathbf{t}_{1}(x) & =x+14 \bmod 26 \\
\mathbf{t}_{2}(x) & =x+19 \bmod 26,
\end{aligned}
$$

i.e.,

A B C D E F GH I J KLMNOPQRSTUVWXYZ HI J K LMNOPQRSTUVWXYZABCDEFG
OPQRSTUVWXYZABCDEFGHI J K LMN TUVWXYZABCDEFGHIJKLMNOPQRS

## Putting it all Together

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$$

i.e.,

A B C D E F GHI J KLMNOPQRSTUVWXYZ HI J K LMNOPQRSTUVWXYZABCDEFG
OPQRSTUVWXYZABCDEFGHI J K LMN TUVWXYZABCDEFGHIJKLMNOPQRS
provide the key word HOT.

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH


Plaintext: T

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH


Plaintext: TH

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH


Plaintext: THE

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH

|  |  | C | D E |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | C | D | E F | G | H | I J | K | L | M | N | $\bigcirc \quad \mathrm{P}$ | Q | R | S | T | U | V | W |  |  | Z |  |
| C | D | E | G | H | I | K | L | M | N | $\bigcirc \mathrm{P}$ | P Q | R | S | I | U | V | W | X | Y |  | A |  |
| D | E | F | G H | I | J | K L | M | N | O | Q | Q R | S | T | U | V | W | X | Y | Z |  | B |  |
| E | F | G | H | J | K | L M | N | O | P | Q R | R | T | U | V | W |  | Y | Z | A |  | C | D |
| F | G | H | I J | K | L | M N | - | $P$ | Q | R | S | U | V | W | X |  | Z | A | B |  |  | E |
| G | H | I | K | L | M | N O | P | Q | R | S T | U | V | W | X | Y |  | A | B | C |  |  |  |
| H | I | J | K | M | N | O | Q | R | S | U | U V | W | X | Y | Z | A | B | C | D |  |  |  |
| I | J | K | L M | N | O | Q | R | S | T | U V | V W | X | Y | Z | A | B | C | D | E |  | G |  |
| J | K | L | M | O | P | Q R | S | T | U V | V W | W | Y | Z | A | B | C | D | E | F |  |  |  |
| K | L | M | N O | P | Q | R S | T | U | V W | W X | X | Z | A | B | C | D | E | F | G |  |  |  |
| L | M | N | O | Q | R | S T | U | V | W | X | Y Z | A | B | C | D | E | F | G |  |  |  |  |
| M | N | O | P Q | , | S | T U | V | W | X | Y Z | Z A | B | C | D | E | F | G | H |  |  |  |  |
| N | O | P | Q R | S | T | U V | W | X | Y | Z | A B | C | D | E | F | G | H |  |  |  |  |  |
| O | P | Q | R | T | U | W | X | Y | Z | A | B C | D | E |  | G | H |  |  | K |  |  |  |
|  | Q | R | S | U | V | W X | Y | Z | A | B | C D | E | F | G | H |  |  | K | L |  |  |  |
| Q | R | S | T | V | W | X Y | Z | A | B | C | D E | F | G | H |  |  | K | L |  |  |  |  |
| R | S | T | U | W | X | Y Z | A | B | C | D | E | G |  |  | J | K | L | M |  |  |  | Q |
|  | T | U | W | X | Y | Z A | B | C | D | E | F G | H |  |  | K |  | M | N |  |  |  |  |
| T | U | V | W X | Y | Z | A B | C | D | E | F | G H |  | J | K | L | M | N | O |  | Q |  |  |
| U | V | W | X Y | Z | A | B C | D | E | F | G | H | J | K | L | M | N | O | P | Q |  |  |  |
| V | W | X | Y Z | A | B | C D | E | F | G | H | I J | K | L | M | N | O | P | Q | R | S |  |  |
| W | X | Y | Z A | B | C | D | F | G | H | I J | K | L | M | N | O | P | Q | R | S |  |  |  |
|  | Y | Z | A | C | D | E | G | H | I | J K | K L | M | N | O | P | Q | R | S | T |  |  |  |
|  | Z | A | B C | D | E | F G | H | I | J | K | L M | N | O |  | Q | R | S | T | U |  |  |  |
|  | A | B | C D |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Plaintext: THES

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH


Plaintext: THEST

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH


Plaintext: THESTO

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH


Plaintext: THESTOV

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH


Plaintext: THESTOVE

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH


## Plaintext: THESTOVEI

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH


Plaintext: THESTOVEIS

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH


## Plaintext: THESTOVEIST

## Deciphering

Keyword: HOT Ciphertext: AVXZHHCSBZHALVXHFMVTLHIGH

|  |  |  | D E |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | C | D | E F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W |  |  | Z |  |
| C | D | E | F G | H | I | J | K | L | M | N | O | P | Q | Q | S | T | U | V | W | X | Y | Z | A | B |
| D | E | F | G H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C |
| E | F | G | H | J | K |  | M | N | O | P | Q |  | S | T | U | V | W | X | Y | Z | A | B | C | D |
| F | G | H | I J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D |  |
| G | H | H | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E |  |
| H | I | J | K L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D |  |  | G |
| I | J | K | M | N | O | P | Q | R | S | T | U | $V$ | W | X | Y | Z | A | B | C | D | E | F | G | H |
| J | K | L | M N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  |
| K | L | M | N O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  |  |
|  | M | N | O | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  |  |  |
| M | N | N O | P Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  |  | K |  |
| N | O | P | Q R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I |  | K |  |  |
| O | P | Q | R | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M |  |
| P | Q | R | S T | U | V W | W | X | Y | Z | A | B | C | D | E | F | G | H | I |  | K | L | M | N |  |
| Q | R | S | T U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |  | J | K | L |  | N |  |  |
|  | S | T | U | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M |  | O |  |  |
|  | T | U | V W | X | Y | Z | A | B | C | D | E | F | G | H |  | J | K | L | M | N | O | P | Q |  |
| T | U | V | W X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R |  |
| U | V | W | X | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S |  |
| $\checkmark$ | W |  | Y Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |  |
| W | X | Y | Z A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V |
|  | Y | Z | A B | C | D | E | F | G | H | I | J | K |  | M | N | O | P | Q | R | S | T |  |  |  |
|  | Z | A | B C | D | E | F | G | H | I | J | K | L |  | N | O | P | Q | R | S | T | U | V |  |  |
|  | A | B | C D |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Plaintext: THESTOVEISTHEHEARTOFSAUNA...

## Inserting Blanks

## THE STOVE IS THE HEART OF SAUNA

## WHEN YOU THROW . . .

## Inserting Blanks

## THE STOVE IS THE HEART OF SAUNA WHEN YOU THROW . . .

Wow, that's a message about sauna.

## Complete Solution

"The stove is the heart of sauna. When you throw water on the stones, the air becomes more humid and feels hotter. You are thus able to experience both dry and humid heat in sauna. The art of sauna building is not discussed here. The most common mistake in building a sauna is to have too small a stove with too few stones. If the stove is only a miserable tiny metal box with a couple of stones on the top, then the room cannot be heated properly unless it is very small. Never be stingy with the heart of sauna."

## Final Remarks

VXH appeared thrice in ciphertext.

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## HEART, HEATING and THEART.

So, there was a bit luck for deciphering the text.
On the other hand, the most frequent letters in each class have been E, T, A, O, N, I, S, H, R.
Thus, the keyword was much too short for the plaintext.

## Final Remarks

Kahn credits Giovan Batista Belaso (1553) for having "proposed the use of a literal, easily remembered, and easily changed key
...for a polyalphabetic cipher," for what we know today as the Vigenère cipher.

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THIS IS A SECRET MESSAGE

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we choose a secret seed key character, say "D," and we write:

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THIS IS A SECRET MESSAGE
we choose a secret seed key character, say " $D$, ," and we write:

| autokey: | DTHISISASECRETMESSAG |
| :--- | :--- |
| message: | THISISASECRETMESSAGE |

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THIS IS A SECRET MESSAGE
we choose a secret seed key character, say "D," and we write:

$$
\begin{array}{ll}
\text { autokey: } & \text { DTHISISASECRETMESSAG } \\
\text { message: } & \text { THISISASECRETMESSAGE }
\end{array}
$$

Additionally, Vigenère proposed scrambling the row and column indexing alphabets at the top and side. This scrambling plus the seed character would form what we would consider the "secret key" nowadays.

## Thank you!



Bruce Schneier


Julius Caesar



David Kahn

