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## 演習第一

1. Let  $X = \mathbb{N}$ . We define  $x\rho y$  if  $x$  divides  $y$ . Prove or disprove
  - (a)  $(\mathbb{N}, \rho)$  is a partially ordered set.
  - (b) The relation  $\rho$  is an equivalence relation.
2. Let  $X$  be any non-empty set and let  $\rho$  be any equivalence relation over  $X$ . Prove or disprove that for all  $x, y \in X$  we have:
  - (a) either  $[x] = [y]$  or  $[x] \cap [y] = \emptyset$ .
  - (b)  $\bigcup_{x \in X} [x] = X$ .
3. Prove or disprove: For every binary relation  $\rho$  over a set  $X$  we have  $\rho^* = (\rho^*)^*$ , i.e., the reflexive–transitive closure of the reflexive–transitive closure is the reflexive–transitive closure itself.