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## 演習第四

1. Prove the following extension of Nerode's theorem:

*Let  $\Sigma$  be any alphabet and let  $L \subseteq \Sigma^*$  be any language. Then we have*

*$L \in \mathcal{REG}$  if and only if there is a right invariant equivalence relation  $\approx$  such that  $\approx$  is of finite rank and  $L$  is the union of some equivalence classes with respect to  $\approx$ .*

2. Prove or disprove:

(1)  $\mathcal{REG}$  is closed under *transposition*, i.e., for all  $L \in \mathcal{REG}$  we have  $L^T \in \mathcal{REG}$ .

(2) There is an algorithm which on input any regular grammars  $\mathcal{G}_1, \mathcal{G}_2$  decides whether or not  $L(\mathcal{G}_1) = L(\mathcal{G}_2)$ .

3. Consider again the language  $L = \{a^n b^n \mid n \in \mathbb{N}\}$ . Use the Nerode Theorem to show that  $L \notin \mathcal{REG}$ .