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コンピューターサイエンス演習 I

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演習第十

1. Let $n \in \mathbb{N}^+$ and let $\mathcal{REG}_n$ be the class of all regular languages that can be accepted by a DFA having at most n states. Furthermore, let $\mathcal{CF}_n$ be the class of all context-free languages that can be accepted via empty stack by a PDA having at most n states.

   Prove or disprove the following assertions:

   (a) $\mathcal{REG}_n \subset \mathcal{REG}_{n+1}$ for all $n \in \mathbb{N}^+$;

   (b) $\mathcal{CF}_n \subset \mathcal{CF}_{n+1}$ for all $n \in \mathbb{N}^+$.

   Does your answer to (a) or (b) change if we fix the alphabet Σ to be $\{a, b\}$?

2. Prove or disprove the following.

   The language $L = \{a^n b^m \mid n,\ m \in \mathbb{N}^+ \text{ and } 1 \leqslant m \leqslant n^2\}$ is context-free.

3. Consider the following grammar $\mathcal{G} = [\{a, b\}, \{\sigma, B, K, S, W\}, \sigma, P]$, where

$$
\begin{aligned}
P \;=\; \{&\sigma \rightarrow SaK,\ aK \rightarrow WbbK,\ aW \rightarrow Wbb,\ SWb \rightarrow SaB, \\
&SWb \rightarrow aB,\ Bb \rightarrow aB,\ BK \rightarrow K,\ BK \rightarrow \lambda\}\ .
\end{aligned}
$$

   (3.1) Determine $L(\mathcal{G})$.

   (3.2) Prove the correctness of the assertion you made in (3.1).

   (3.3) Prove or disprove $\mathcal{G}$ to be context-sensitive.

4. **Bonus problem:** Let the following pushdown automaton $\mathcal{K} = [\{q_0, q_1, q_2\}, \{0, 1\}, \{0, 1\}, \delta, q_0, 1, \{q_2\}]$ be given, where

$$
\delta(q_i, x, y) = \begin{cases} (q_0, xy)\ , & \text{if } i = 0,\ x = 0,\ y \in \{0, 1\}\ ; \\ \{(q_1, \lambda), (q_2, \lambda)\}\ , & \text{if } i \in \{0, 1\},\ x = 1,\ y \in \{0, 1\}\ . \end{cases}
$$

   Provide a context-free grammar $\mathcal{G}$ for $L(\mathcal{K})$ by using the construction given in the proof of Theorem 9.3.